1.6 Calculating Displacement During Accelerated Motion

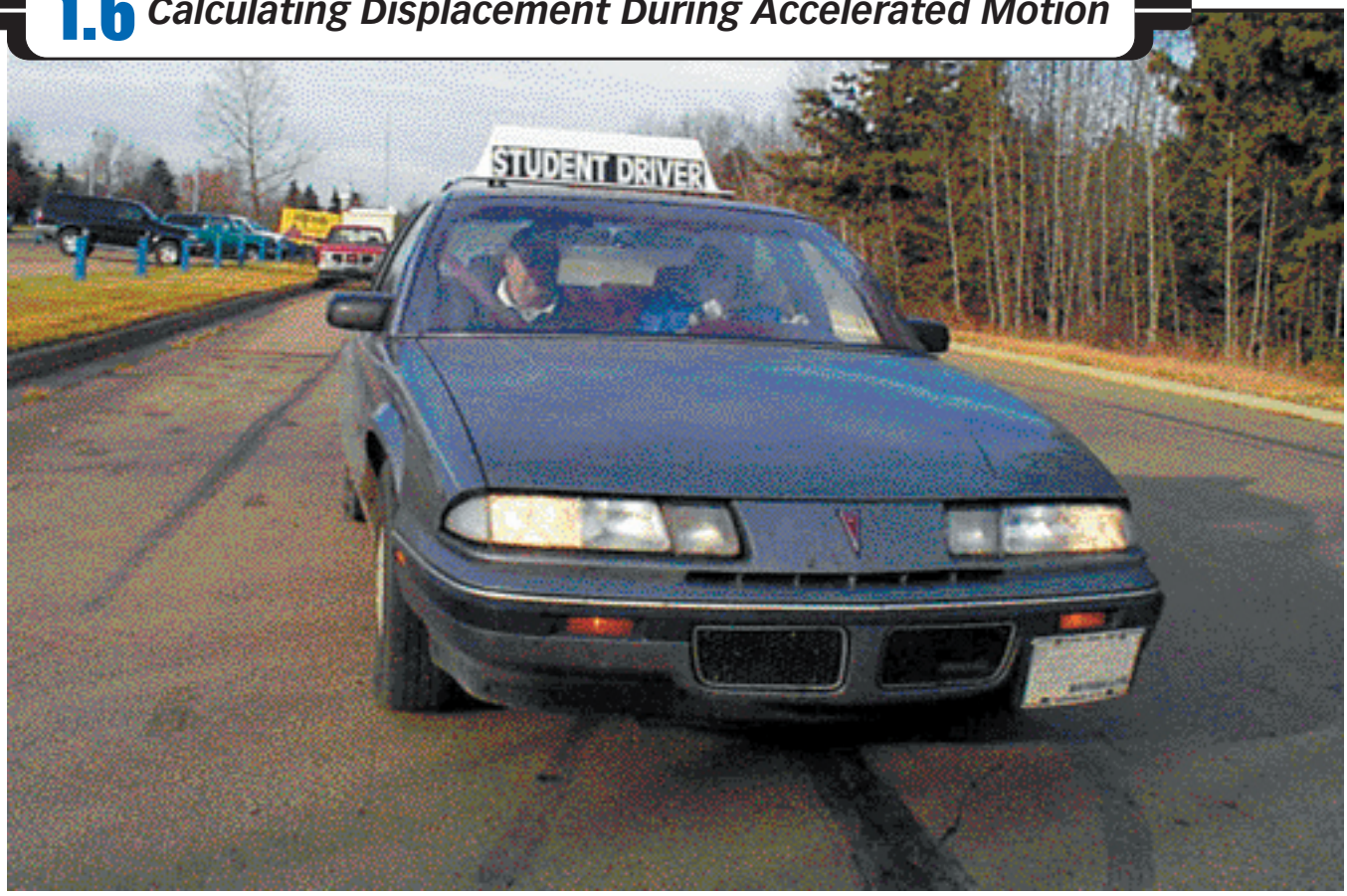


Figure B1.40: It is a good idea to take driver training before getting your driver's licence.

If you've studied the *Basic Licence Driver's Handbook* and taken the "Rules of the Road" test, you would qualify for your learner's licence. This licence means exactly what the name implies—you now have the privilege of learning to drive under the direction of a driving coach, who will help you develop the skills to become a safe driver.

The first driving lesson usually deals with learning the vehicle's instrument panel and controls; then you progress to starting and parking, followed by driving in residential areas. Once you have mastered these skills, your driving coach will prepare you for highway driving. It can be a little nerve-wracking the first time, especially when you enter the on-ramp and prepare to merge into dense traffic already travelling highway speeds.

Drivers already on a highway often see a sign like the one on the right to remind them that merging traffic may be entering the highway. When it is possible and safe to do so, drivers on the highway should move to the left lane to clear the right lane for merging traffic to enter. If it is not possible to move to the left, vehicles on the highway should co-operate by creating safe gaps for the merging vehicles.

DID YOU KNOW?

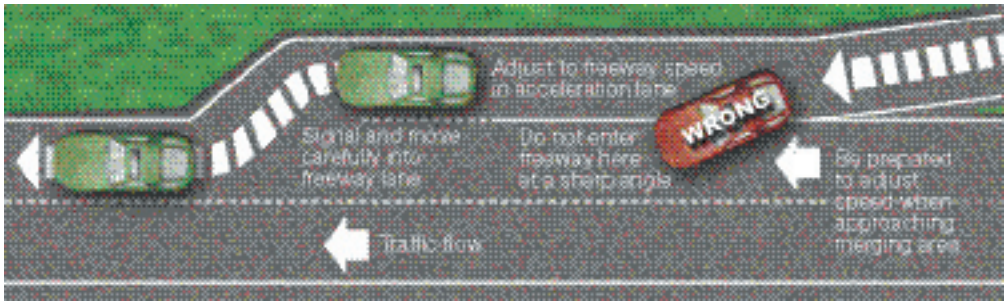
It is against the law to cross over the solid line that divides the acceleration lane from the other lanes of traffic on a highway. Drivers should use most of the acceleration lane to adjust to highway speed before merging safely into a gap in the traffic. This ensures that the merging vehicle is crossing only the broken lines on the highway that occur at the end of the acceleration lane.

Practice

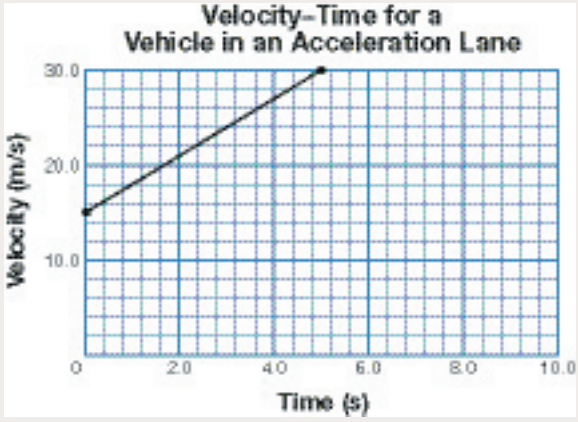
Use the following information from a driver training handbook regarding merging to answer questions 30 and 31.

Merging

Merging requires that you time your approach and smoothly blend in with the other traffic. This may require adjusting your speed so that when you reach the end of the acceleration lane, you have a gap in the traffic, which will permit you to enter the flow of traffic safely. Your entry into that flow should be at, or near, the speed of the other traffic.


Adjust to freeway speed in acceleration lane
Signal and move carefully into freeway lane
Do not enter freeway here at a sharp angle
WRONG
Be prepared to adjust speed when approaching merging area
Traffic flow

30. Why is the first lane you enter on a highway called the acceleration lane?
31. The following graph represents typical data for a vehicle in an acceleration lane.



a. Determine the initial velocity and the final velocity of the vehicle in metres per second and kilometres per hour.
b. Using the data on the graph, calculate the magnitude of the acceleration of this vehicle in m/s^2.
c. If you were travelling in another vehicle that was underpowered, the acceleration could be less than the value you calculated in question 31.b. If the initial velocity and the final velocity were still the same, describe how the shape of the graph would be different.
d. Explain why it's important for the acceleration lane to be long enough to accommodate a wide range of possible vehicle accelerations.

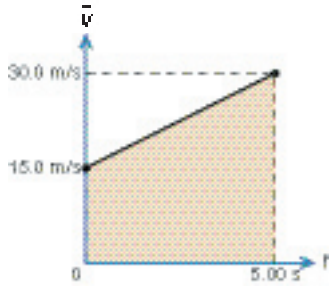
Traffic Safety Engineering

A freeway acceleration lane is a very carefully designed structure. It is critical that this lane be the right length to accommodate the wide range of vehicles that will use this entrance to access the highway. Who decides how long these lanes should be? How do they decide how long this lane should be? Is it the same length for every highway? Given the information presented in the previous Practice questions, you should be able to determine the minimum length of the acceleration lane. Although professional engineers would use a more in-depth approach, the concepts of displacement, velocity, and acceleration are important variables in their calculations.

Determining the Minimum Length of an Acceleration Lane

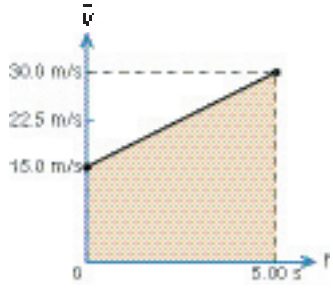
An acceleration lane must accommodate a vehicle from the time it enters with its initial velocity to the time it leaves with its final velocity. So, it makes sense to start by calculating the displacement of the vehicle while it accelerates. Since the displacement is the area under a velocity–time graph, the velocity–time graph will have to be analyzed.

step 1: Set up the velocity–time graph to include labels for initial velocity (15.0 m/s), final velocity (30.0 m/s), and time (5.00 s).

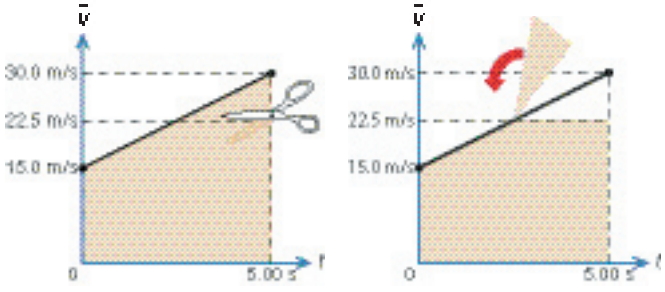


step 2: Calculate the average of the two velocities, and add this value to the graph.

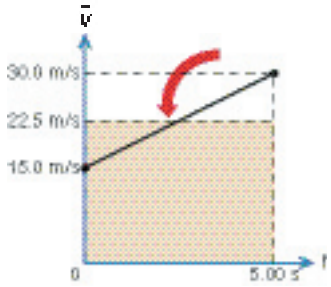
$$\text{average velocity} = \frac{\vec{v}_i + \vec{v}_f}{2}$$
$$= \frac{(+15.0\ \text{m/s}) + (+30.0\ \text{m/s})}{2}$$
$$= +22.5\ \text{m/s}$$



step 3: Cut the graph horizontally through the average velocity value.



step 4: Take the top, cut-out portion of the graph and fit it into the empty part of the graph to create a rectangular shape.



step 5: Use substitution to determine the displacement. Since the displacement is equal to the area under the graph, the displacement should be equal to the area of the rectangle (length ¥ width), which is average velocity multiplied by time.

$$\text{displacement} = \text{area under graph}$$
$$\Delta\vec{d} = l \times w$$
$$= (\text{average velocity})(\text{time})$$
$$= (+22.5\ \text{m/s})(5.00\ \text{s})$$
$$= +113\ \text{m}$$

step 6: Express displacement in terms of final and initial velocities. Notice that adding the initial velocity to the final velocity and dividing by 2 will always equal the average of the two velocities.

$$\text{displacement} = (\text{average velocity})(\text{time})$$
$$\Delta\vec{d} = \left(\frac{\vec{v}_i + \vec{v}_f}{2}\right)\Delta t$$

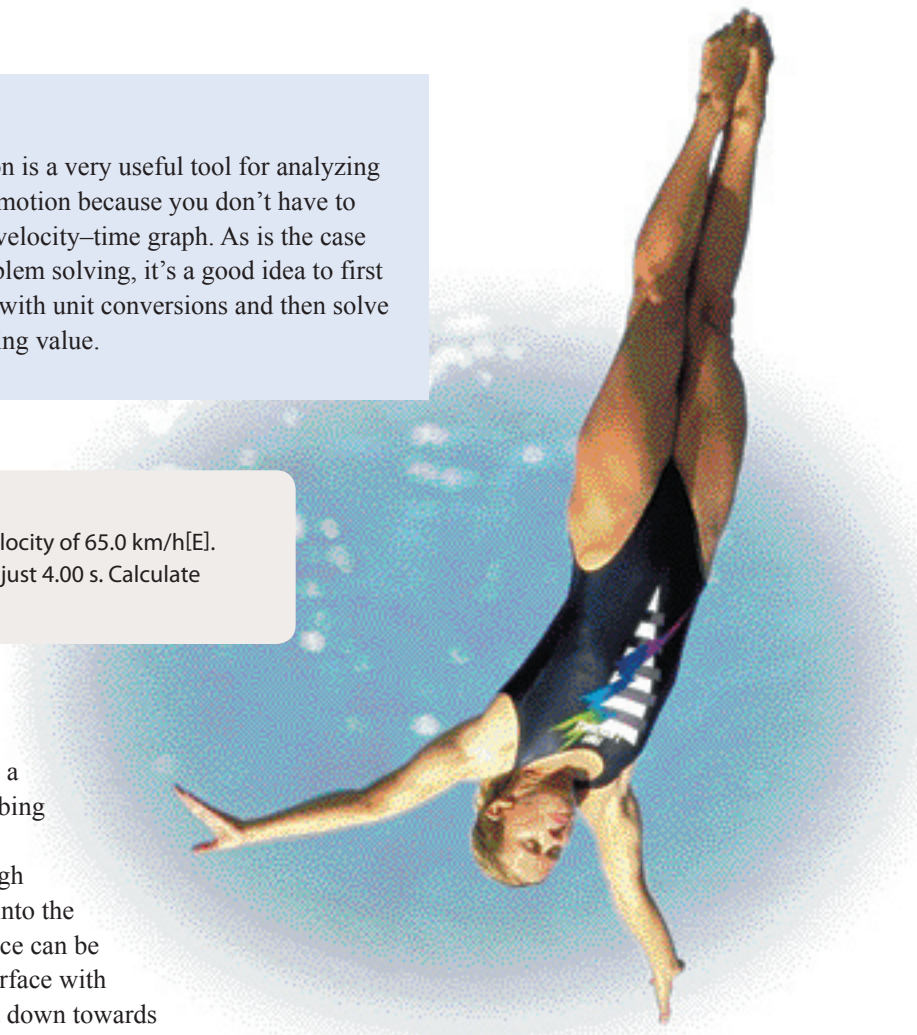
Using the Displacement Equation

initial velocity — final velocity

$$\Delta\vec{d} = \left(\frac{\vec{v}_i + \vec{v}_f}{2}\right)\Delta t$$

displacement — time interval

This equation is a very useful tool for analyzing accelerated motion because you don't have to start with a velocity–time graph. As is the case with all problem solving, it's a good idea to first list the data with unit conversions and then solve for the missing value.



Practice

32. A car enters the acceleration lane with an initial velocity of 65.0 km/h[E]. The car reaches a final velocity of 100.0 km/h[E] in just 4.00 s. Calculate the displacement of the car in metres.

Acceleration Due to Gravity

The displacement equation is not only useful for solving problems involving the motion of vehicles on a highway, it can be applied in any circumstance describing accelerated motion. Some of the most interesting examples have to do with the motion of objects through the air, such as a person diving from a high platform into the pool. If the distances are not too great and air resistance can be ignored, the diver falls through the air near Earth's surface with an average acceleration of 9.81 m/s^2, directed straight down towards the centre of the planet. When problem solving, this value is sometimes not included with the data given because it is expected that you will know this value and know when to use this value. Problems involving objects that move through the air near Earth's surface frequently involve displacement, velocity, and acceleration vectors that do not all point in the same direction. In these cases, the vector nature of these quantities must be considered.

Example Problem 1.15

A baseball leaves a bat and travels straight up into the air, reaching its highest point 15.9 m above the bat in just 1.8 s.

a. Determine the initial velocity of the ball using the displacement equation.
b. Verify your answer by calculating the initial velocity using another equation.

Solution

a.

$\Delta\vec{d} = 15.9\text{ m[up]}$
$= +15.9\text{ m}$
$\Delta t = 1.8\text{ s}$
$\vec{v}_f = 0$ — The ball stops at its highest point, making $\vec{v}_f = 0$.
$\vec{v}_i = ?$

$$\Delta\vec{d} = \left(\frac{\vec{v}_i + \vec{v}_f}{2}\right)\Delta t$$

$$\Delta\vec{d} = \left(\frac{\vec{v}_i}{2}\right)\Delta t$$

$$\vec{v}_i = \frac{2\Delta\vec{d}}{\Delta t}$$

$$= \frac{2(+15.9\text{ m})}{1.8\text{ s}}$$

$$= +18\text{ m/s}$$

The initial velocity of the ball is 18 m/s[up].

b.

$\vec{a} = 9.81\text{ m/s}^2\text{ [down]}$
$= -9.81\text{ m/s}^2$
$\Delta t = 1.8\text{ s}$
$\vec{v}_f = 0$
$\vec{v}_i = ?$

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\vec{a} = \frac{-\vec{v}_i}{\Delta t} \quad \leftarrow \vec{v}_f = 0$$

$$-\vec{v}_i = \vec{a}\Delta t$$

$$\vec{v}_i = -\vec{a}\Delta t$$ — Multiply both sides by -1.

$$= -\left(-9.81\text{ m/s}^2\right)(1.8\text{ s})$$

$$= +18\text{ m/s}$$

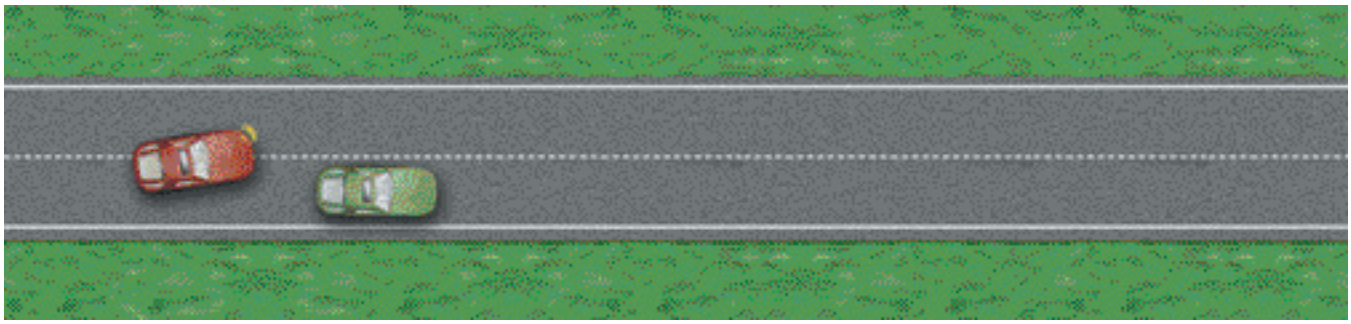
The initial velocity of the ball is 18 m/s[up].

Practice

33. During a volleyball tournament, a player dives to prevent the ball from hitting the floor. The ball leaves the outstretched player's arms with an initial velocity of 14.5 m/s, directed straight up, and stops just below the ceiling, 10.7 m above the player's arms.
    a. Calculate the time required for the ball to travel from the player's arms to the ceiling using the displacement equation.
    b. Verify your answer by calculating the time using another equation.

Passing Another Vehicle on the Highway

Many young drivers are surprised the first time they pass another vehicle on the highway by the time and distance required to complete the pass. If the highway is a two-lane highway with one lane of traffic going in each direction, errors in judgement could place drivers in a situation that results in a head-on collision. The next example problem illustrates the distance travelled by a vehicle during a typical passing manoeuvre.



Example Problem 1.16

A car travelling 90 km/h accelerates at 0.50 m/s^2 while passing another vehicle. If it takes 5.0 s to pass the vehicle, determine the distance travelled by the vehicle during this time.

Solution

Let the velocity, acceleration, and displacement vectors all point in the positive direction.

$$\vec{v}_i = +90\frac{\text{km}}{\text{h}} \times \frac{1000\ \text{m}}{1.0\ \text{km}} \times \frac{1\ \text{h}}{3600\ \text{s}}$$
$$= +25\ \text{m/s}$$

$$\vec{a} = +0.50\ \text{m/s}^2 \qquad \Delta\vec{d} = ?$$
$$\Delta t = 5.0\ \text{s}$$

$$\Delta\vec{d} = \left(\frac{\vec{v}_i + \vec{v}_f}{2}\right)\Delta t$$

Solving for $\Delta\vec{d}$ requires that you first find $\vec{v}_f$.

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$
$$\vec{v}_f = \vec{a}\Delta t + \vec{v}_i$$
$$= (+0.50\ \text{m/s}^2)(5.0\ \text{s}) + (+25\ \text{m/s})$$
$$= +27.5\ \text{m/s}$$

Now, solve for $\Delta\vec{d}$.

$$\Delta\vec{d} = \left(\frac{\vec{v}_i + \vec{v}_f}{2}\right)\Delta t$$
$$= \frac{(+25\ \text{m/s}) + (+27.5\ \text{m/s})}{2}(5.0\ \text{s})$$
$$= +131\ \text{m}$$
$$= +1.3 \times 10^2\ \text{m}$$

The car travels 1.3 ¥ 10^2 m while passing.

Building a New Equation

In Example Problem 1.16, the calculation of the distance travelled by an accelerating vehicle became quite complex because the final velocity was not given in the original data. Problems like this are so common that it is worthwhile to develop a new equation to simplify solving situations just like this. The following steps show how a new equation can be developed from a velocity–time graph describing Example Problem 1.16.

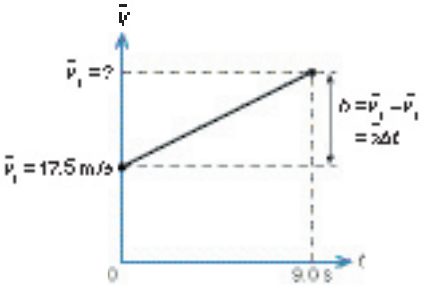
step 1: Rearrange the acceleration equation to isolate $(\vec{v}_f - \vec{v}_i)$. This quantity will correspond to a key feature in the graphs shown in the next two steps.

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

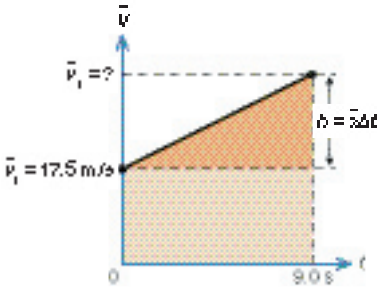
$$\vec{a}\Delta t = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}\Delta t \quad \text{Multiply both sides by } \Delta t.$$

$$\vec{a}\Delta t = \vec{v}_f - \vec{v}_i$$

step 2: The velocity–time graph includes labels for the initial velocity, 17.5 m/s, and the time, 9.0 s. To include the acceleration value as a label on this graph, notice that the height of the triangular area of the graph is equal to $(\vec{v}_f - \vec{v}_i)$. This is also equal to the acceleration multiplied by time (as shown in step 1).



step 3: Displacement is equal to the area under the velocity–time graph. As you can see, this is equal to the sum of the area of a rectangle and the area of a triangle.



displacement = area under graph

$$\Delta\vec{d} = \text{area of rectangle} + \text{area of triangle}$$

step 4: Recall that the area of a rectangle is equal to the length times the width, A = *l* ¥ *w*, and that the area of a triangle is equal to one-half the base times the height, $A = \frac{1}{2}(b \times h)$. Apply these formulas, and substitute the variables from the graph.

$$\Delta\vec{d} = (l \times w) + \frac{1}{2}(b \times h)$$

$$= (\vec{v}_i)(\Delta t) + \frac{1}{2}(\Delta t)(\vec{a}\Delta t)$$

$$= \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

Substitute the values from Example Problem 1.16 into the equation shown in step 4 to confirm that this method can be used to calculate displacement.

$\vec{v}_i = +25.0$ m/s $\qquad \Delta t = 5.0$ s

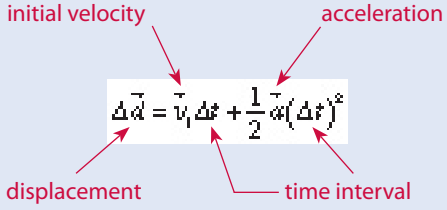
$\vec{a} = +0.50$ m/s $\qquad \Delta\vec{d} = ?$

$$\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

$$= (+25.0 \text{ m/s})(5.0 \text{ s}) + \frac{1}{2}(+0.50 \text{ m/s}^2)(5.0 \text{ s})^2$$

$$= +1.3 \times 10^2 \text{ m}$$

Using Another Displacement Equation



This equation allows for a simpler, more direct approach than the method presented in Example Problem 1.16. The only caution is that you have to be careful to remember to square the time interval in the second half of the equation.

Example Problem 1.17

Juanita leaves the surface of a trampoline with an initial velocity of 11.8 m/s, directed straight up. Determine the displacement of the gymnast after 0.80 s.

Solution

$\vec{a} = 9.81\ m/s^2\ [down]$
$= -9.81\ m/s^2$

Note: The data may seem incomplete; but, in actuality, the gymnast is under the influence of gravity.

$\vec{v}_i = 11.8\ m/s\ [up]$
$= +11.8\ m/s$

$\Delta t = 0.80\ s$

$\Delta \vec{d} = ?$

$$\Delta \vec{d} = \vec{v}_i \Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$
$$= (+11.8\ m/s)(0.80\ s) + \frac{1}{2}(-9.81\ m/s^2)(0.80\ s)^2$$
$$= +6.3\ m$$

The gymnast is 6.3 m above the surface of the trampoline.

Example Problem 1.18

A diver steps off the edge of a platform and enters the water 5.0 m below. If the initial velocity of the diver was zero, determine the time for it took the diver to reach the water.

Solution

Since solving for time requires you to divide the displacement by the acceleration, the vector notation is dropped.

$\Delta d = 5.0\ m\ [down]$
$= -5.0\ m$

$a = 9.81\ m/s^2\ [down]$
$= -9.81\ m/s^2$

$v_i = 0$

$\Delta t = ?$

$$\Delta d = v_i \Delta t + \frac{1}{2} a (\Delta t)^2$$

$$\Delta d = \frac{1}{2} a (\Delta t)^2 \quad \leftarrow v_i = 0$$

$$\Delta t^2 = \frac{2\Delta d}{a}$$

$$\Delta t = \sqrt{\frac{2\Delta d}{a}}$$
$$= \sqrt{\frac{2(-5.0\ m)}{-9.81\ m/s^2}}$$
$$= 1.0\ s$$

It took the diver 1.0 s to reach the water.

Practice

34. A vehicle travelling 63 km/h[E] accelerates 1.0 m/s^2[E] for 9.0 s. Determine the displacement of the vehicle during this 9.0-s time interval.
35. The driver of a vehicle travelling 84 km/h on a country road is startled to see a deer standing in the middle of the road. By applying the brakes, the driver is able to slow down at a rate of -4.5 m/s^2 while sounding the horn. Luckily, the deer bolts before a collision occurred. Exactly 5.0 s passes from the moment the brakes are applied to the instant the deer bounds from the road. Calculate the displacement of the vehicle during the 5.0-s time interval.

Deceleration

deceleration: the slowing down of an object due to the acceleration vector pointing in the opposite direction as the velocity vector

In Practice question 35, the vehicle had a positive initial velocity and a negative acceleration. This meant that the magnitude of the velocity vector would be reduced and the vehicle would slow down. In the context of vehicles moving on a roadway, this might be referred to in everyday language as **deceleration**. Although this term is frequently used to describe the motion of vehicles on a roadway, it is not a term that is generally used in pure science applications. If you encounter a problem that describes the motion of a vehicle moving in the positive direction as "decelerating at 4.5 m/s^2," you could translate this to mean a negative acceleration, -4.5 m/s^2.

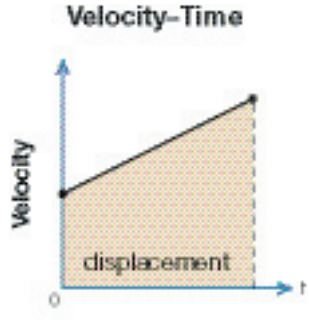
1.6 Summary

Displacement can be calculated using the area under a velocity–time graph. Finding the area under a velocity–time graph results in two new equations. Each of these equations can be rearranged to solve for the other variables.

Again, since it goes beyond high school mathematics to multiply or divide two vectors, the vector notation is dropped when solving for the time interval, D*t*.

If air resistance can be ignored when an object falls through the air near Earth's surface, it accelerates at 9.81 m/s^2, directed straight down toward the centre of Earth. The displacement equations can be used to solve problems when objects fall through the air; but special care must be taken with the proper use of the sign convention.

Deceleration is a term that refers to the slowing down of an object due to the acceleration vector pointing in the direction opposite to the velocity vector. When solving problems with positive velocity vectors, it is preferable to translate this to mean a negative acceleration.



$$\Delta\vec{d} = \left(\frac{\vec{v}_i + \vec{v}_f}{2}\right)\Delta t$$

If $\vec{v}_i = 0$,

$$\Delta\vec{d} = \left(\frac{\vec{v}_f}{2}\right)\Delta t$$

$$\vec{v}_f = \frac{2\Delta\vec{d}}{\Delta t}$$

$$\Delta t = \frac{2\Delta d}{v_f}$$

If $\vec{v}_f = 0$,

$$\Delta\vec{d} = \left(\frac{\vec{v}_i}{2}\right)\Delta t$$

$$\vec{v}_i = \frac{2\Delta\vec{d}}{\Delta t}$$

$$\Delta t = \frac{2\Delta d}{v_i}$$

$$\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

If $\vec{v}_i = 0$,

$$\Delta\vec{d} = \frac{1}{2}\vec{a}(\Delta t)^2$$

$$\vec{a} = \frac{2\Delta\vec{d}}{(\Delta t)^2}$$

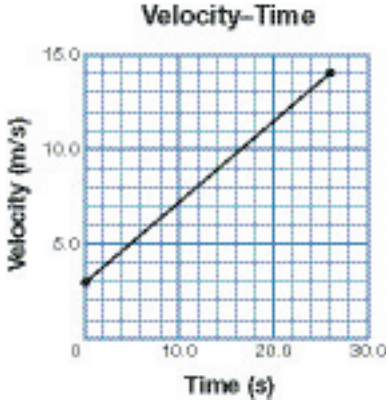
$$\Delta t = \sqrt{\frac{2\Delta d}{a}}$$

1.6 Questions

Knowledge

1. The following graph is typical for a car speeding up with uniformly accelerated motion. Copy this graph into your notebook, and add the following labels.

- $\vec{v}$
- $\vec{v}_f$
- Δt
- $(\vec{v}_f - \vec{v}_i)$
- $\vec{a}\Delta t$



2. Refer to the graph in question 1.

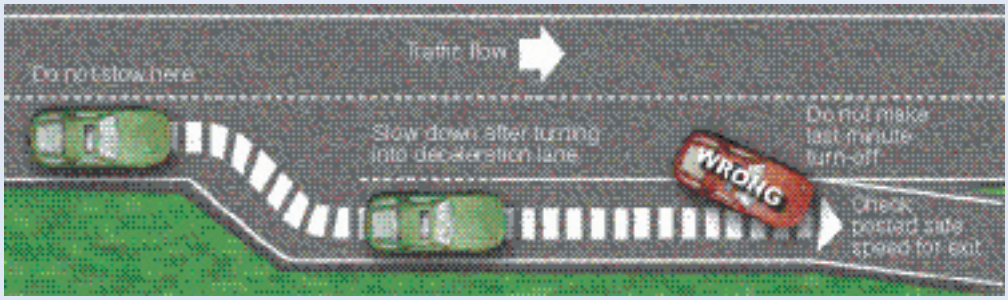
a. Create a detailed description showing the origins of the equation $\Delta\vec{d} = \left(\frac{\vec{v}_i + \vec{v}_f}{2}\right)\Delta t$. Your flowchart should show how the labelled variables from the graph are assembled to create this equation.

b. Create a detailed description showing the origins of the equation $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$ from a velocity–time graph. Your flowchart should show how the labelled variables from the graph are assembled to create this equation.

Applying Concepts

Use the following information to answer question 3.

Just as an acceleration lane provides a safe way to enter a highway, a lane called the deceleration lane provides a vehicle with a safe way to exit a highway. The following diagram shows the layout of a typical deceleration lane for a major highway.


Traffic flow
Do not slow here
Slow down after turning into deceleration lane
Do not make last minute turn-off
WRONG
Check posted safe speed for exit

3. Suppose a vehicle enters the deceleration lane at 95.0 km/h and must slow down to 50.0 km/h before travelling around the curve of the exit ramp. Assume that most passenger cars take 5.00 s to comfortably decelerate in this situation.

a. Determine the vehicle's deceleration.

b. Use the initial velocity, final velocity, and value for time to determine the displacement of the vehicle during the 5.00-s interval.

c. Use the initial velocity, value for time, and your value for the deceleration from question 3.a. to calculate the displacement of the vehicle during the 5.00-s interval.

d. Compare your answers to questions 3.b. and 3.c. Are they the same? What do your answers suggest about the minimum length of the deceleration lane for this case?

e. If the highway were wet, a vehicle might only be able to manage a deceleration of 1.50 m/s^2 as it slows down from 95.0 km/h to 50.0 km/h. Determine the minimum length for the deceleration lane for this case.

f. Compare your answers to questions 3.d. and 3.e. Explain why the total length of the deceleration lane would probably be designed to be more than 170 m long for this exit.

4. During a baseball game, a ball leaves a bat and travels straight up into the air. A TV camera records that when the ball reaches its highest point, it had travelled 18.5 m from where it left the bat.

a. Consider the part of the ball's motion when it falls from its highest point back down toward the batter. Determine the time it takes the ball to travel this distance.

b. Ignoring air resistance, when things travel through the air, their motion is symmetrical—the second half of motion mirrors the first half of motion. Use this idea to determine the time for the ball to travel from its release point on the bat to its highest point.

c. Use your answer to question 4.b. to determine the magnitude and direction of the initial velocity of the ball immediately after it leaves the bat.